for the last five years in Brazil, and who by my request has given especial attention to the matter, is the immediate occasion of my inquiry. Protective mimicry is a fact too well established to admit of its supporters feeling the question a delicate one. Admission into your paper will speedily settle the question.

Liverpool, Eebruary 2 HENRY H. HIGGINS

ON THE GRADUATION OF GALVANOMETERS FOR THE MEASUREMENT OF CURRENTS AND POTENTIALS IN ABSOLUTE MEASURE[1]

IV.

WE shall now consider, very briefly, the graduation of instruments for measuring volts and amperes in practical work, and we shall take as our example Sir William Thomson's graded galvanometers. The graduation of these instruments is effected by a comparison of their indications with those of a standard galvanometer such as that described above. We shall consider first the graduation of a potential galvanometer, or galvanometer the resistance of which is so high, that the attachment of its terminals to two points in a conductor carrying a current does not perceptibly change the difference of potentials formerly existing between these points. Of course any galvanometer which measures currents also measures potentials, for, if its resistance is known, the difference of potentials between its terminals can be calculated from Ohm's law; but the convenience of a galvanometer specially made with a high resistance coil is that the difference of potentials, thus calculated as existing between the two points at which its terminals are applied while they are in contact, may be taken as the actual difference of potentials which exists between those two points when nothing but the ordinary conductor connects them. For, let V be this actual difference of potentials in volts, let r ohms be the resistance of the conductor, and R ohms the resistance of the galvanometer. Then by the application of R, V is diminished in the ratio of R to $R + r$, and therefore the difference of potentials between the ends of the coil is now $V\dfrac{R}{R+r}$. Hence by Ohm's law we have for the current through the galvanometer the value $\dfrac{V}{R}\dfrac{R}{R+r}$, or $\dfrac{V}{R\left(1+\dfrac{r}{R}\right)}$. If r be only a small fraction of R, $\dfrac{r}{R}$ is inappreciable, and the difference of potentials calculated from the equation $C = \dfrac{V}{R}$ will be nearly enough the true value.

The instrument to be graduated is first tested as to the adjustment of its coil, needle, &c. The standard galvanometer and it are then properly set up with their needles pointing to zero, in positions near which there is no iron, and at which the values of H have been determined. The high resistance coil of the standard galvanometer and the coil of the potential instrument are joined in series with a constant battery of as many Daniell's cells as gives a deflection of about 45° on the standard galvanometer, and the magnetometer is adjusted with its index at zero, in such a position on its platform that a deflection of its needle also of nearly 45° is produced. The current actually flowing in the circuit is calculated by equation (11) or (12) from the reading obtained on the standard, and reduced to amperes by multiplying the result by 10. The difference of potentials between the two ends of the coil of the potential galvanometer is found in volts by multiplying the number of amperes thus found by the resistance of the coil in ohms. We can then obtain, by an obvious calculation, the number of divisions of deflection which corresponds to one volt between the two ends of the coil, and thence from the value of H the number of divisions which would correspond to one volt if the intensity of the field were one c.g.s. unit. This would be the number which would be marked at that position on the platform of the instrument; but, except for the position of the magnetometer nearest to the coil, positions the corresponding numbers of which are multiples and submultiples of 2 are alone marked. The numbers corresponding to the two of these positions adjacent on the two sides of the position of the magnetometer in our experiment, may be readily found by keeping the same difference of potential on the coil, and moving the magnetometer nearer to or further from the coil until the deflection is increased or diminished in the proper ratio. For example, let the deflection be 40 divisions for 20 volts, and let the value of H at the instrument which is being graduated be ·17. The number of divisions of deflection which would correspond to one volt for that position, if the field were of unit intensity, would be $\frac{40}{20} \times {\cdot}17 = {\cdot}34$. Hence the marked positions nearest to this in the two sides of it, are to be those for which the corresponding numbers are $\frac{1}{2}$ and $\frac{1}{4}$. Therefore the magnetometer must be moved further from the coil for the latter until the deflection produced by 20 volts becomes 29·4. This is the position at which the number $\frac{1}{4}$ is to be marked. To find the position at which $\frac{1}{2}$ should be marked a smaller difference of potentials must be used, as the deflection with the same battery as before would be beyond the limits of the scale. Suppose that when its number of cells is diminished to one half; we get a deflection for our first position of 20. While the potential difference in the coil remains constant, the magnetometer is pushed in until the deflection again becomes 29·4. At this position the number $\frac{1}{2}$ is to be marked. From this it is easy to see how the position for the number 1 can be found, and in the same way those for the other numbers of the series, 2, 4, &c. The number corresponding to the position of the magnetometer nearest to the coil, although not one of the terms of this series, is determined in the same way, and marked at that position on the platform. This is the method adopted in practice in the graduation of these instruments.

Another method sometimes convenient is as follows. The standard instrument, a few good Daniell's cells, and a resistance which gives a deflection of about 45° on the standard are joined in series, and the galvanometer to be graduated is applied at two points in the circuit which include between them such a portion of the resistance as gives a deflection of about the same amount. Let R ohms be the portion of the resistance included between the terminals of the galvanometer, and let G ohms be the resistance of the galvanometer coil. Let the current calculated from the deflection on the standard be C amperes, then if V be the potential difference in volts between the terminals of the potential instrument, we have by Ohm's law—

$$C = \frac{V}{R'},$$

where R' is the resistance equivalent to the divided circuit of R and G. But $R' = \dfrac{RG}{R+G}$, and therefore

$$C = V\frac{R+G}{RG}.$$

Hence,

$$V = C\frac{RG}{R+G}.$$

This last equation gives the number of volts indicated by the deflection on the potential instrument, for the position at which its magnetometer is placed; and from this in precisely the same manner as described above, the series of positions of the magnetometer on its platform are determined and numbered.

[1] Continued from p. 321.

For verifying the accuracy of the graduation of the potential instruments when performed by either of these methods, a standard Daniell's cell of the form proposed by Sir William Thomson at the Southampton meeting of the British Association is used. It is represented in the annexed cut (Fig. 4). It consists of a zinc plate at the bottom of the vessel resting in a stratum of saturated zinc sulphate, on which has been poured, so gently as to give a clear surface of separation, a stratum of half saturated sulphate of copper solution, in which is immersed a horizontal plate of copper. The copper-sulphate solution is introduced by means of the glass tube shown in the diagram dipping down into the liquid, and terminating in a fine point, which is bent into a horizontal direction so as to deliver the liquid with as little disturbance as possible. This tube is connected by a piece of indiarubber tubing with a funnel, by the raising or lowering of which the sulphate of copper can be run into or run out of the cell. By this means the sulphate of copper is run in when the cell is to be used, and at once removed when the cell is no longer wanted.

The electro-motive force of this cell has been determined very carefully and found, according to Lord Rayleigh's latest determination of the ohm, to be 1·07 volt at ordinary temperatures. The direct application of this cell to the galvanometer gives at once a check on the graduation. As the resistance of the galvanometer is always over 6,000 ohms, there is practically no polarization.

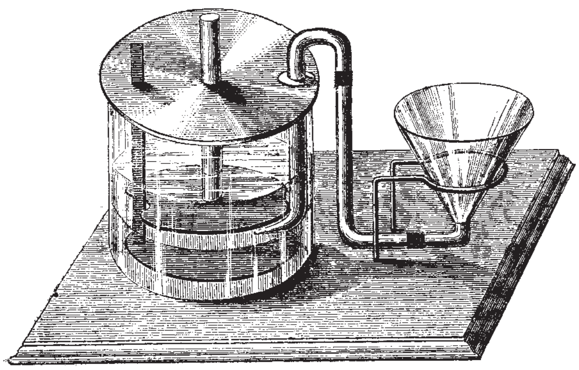

FIG. 4.

The method adopted for the graduation of the current galvanometer is precisely the same as that first described for the potential instrument. The standard galvanometer, of which in this case the low resistance coil is used, and the current galvanometer to be graduated are joined in series with a battery, which with some resistance in circuit is sufficient to give a deflection in each of about 45°, when the magnetometer of the current instrument is at a convenient position on its platform. The current flowing in amperes is given by the standard, and this, of course, is the number of amperes which is indicated by the deflection of the current instrument. By an obvious calculation from the value of H at the current instrument, precisely similar to that above described, the number of divisions of deflection corresponding to a current of one ampere for a field of unit strength is found, and from this the series of positions on the platform and their numbers are found.

The value of the field intensity given at the needles of the magnetometer when in position has generally been determined in the following manner. A battery of about 30 of Sir William Thomson's Tray Daniell's is joined in series with a resistance of about 7,000 ohms. The electrodes of a potential galvanometer, on which the magnetometer is placed without its magnet, are attached at two such points in this resistance that the deflection of the needle produced is from 30 to 40 division on the scale. The current through the galvanometer is now stopped and the magnet placed in position, and the index brought to zero by turning the magnet by means of its screw. The electrodes are now placed so as to include a resistance which makes the deflection nearly what it was in the former case. Let E be the electromotive force of the battery, I the intensity of the horizontal component of the field produced at the needles by the magnet; R_1 R_2 the amount of this resistance included between the electrodes of the galvanometer in the first and second cases respectively; V_1 and V_2 the potential difference in volts on the instrument in the same two cases; D_1 and D_2 the corresponding deflections, and G the resistance of the galvanometer. We have by Ohm's law

$$V_1 = \frac{E R_1 G}{(B + R - R_1)(R_1 + G) + R_1 G} = m H D_1$$

and

$$V_2 = \frac{E R_2 G}{(B + R - R_2)(R_2 + G) + R_2 G} = m I D_2$$

where m is a constant. Therefore we have

$$I = H \frac{D_1 R_2 \{(B + R - R_1)(R_1 + G) + R_1 G\}}{D_2 R_1 \{(B + R - R_2)(R_2 + G) + R_2 G\}}$$

If the resistance B of the battery be small in comparison with G, or if the galvanometer is sensitive enough to allow $\frac{B}{G}$ to be made sufficiently small by resistance added to G, B may be neglected; and it is generally possible, by properly choosing R, R_1, R_2, to simplify very much this formula. The number I thus found, diminished by H, is the number of c.g.s. units which measures the horizontal intensity of the magnetic field produced at the needle by the action of the magnet alone. The value of $I - H$ is the number painted on the magnet, and is generally about 9 or 10.

$I - H$ may be determined by means of a current galvanometer very easily by keeping a constant current flowing through the instrument, and using without the magnet one of the less sensitive positions of the magnetometer, and with the magnet one of the more sensitive positions. If D_1 and D_2 be the deflections and n_1 n_2 the number of divisions of deflections corresponding to one ampere at the two respective positions, the value of I is at once found from the obvious equation

$$\frac{D_1}{n_1 H} = \frac{D_2}{n_2 I}, \text{ or, } I = \frac{n_1 D_2}{n_2 D_1} H.$$

When either instrument has been graduated for a field of intensity equal to 1 c.g.s. unit, and the intensity of the field given by the magnet at the needles has been determined, the graduation of the instrument is complete. In the practical use of the instrument with the magnet in position, the number of volts, or the number of amperes (according as the instrument used is a potential or a current galvanometer) corresponding to a deflection of any number of divisions is found by the following rule:—

Multiply the number of divisions in the deflection by the number on the magnet increased by the horizontal intensity of the earth's field,[1] *and divide by the number at the division on the platform scale exactly under the front of the magnetometer.*

When the magnet is not used the rule is the same as the above, except that the divisor to be used is the value of H for the place of the galvanometer.

For convenience in the ordinary use of either instrument a position of the magnetometer on its platform, which may not be one of the series described above, is determined at which the deflection with the magnet in position, for one volt or one ampere, is one or some other convenient number of divisions. For this position lines

[1] The mean value of H for Great Britain may, *when the magnet is used*, be taken with sufficient accuracy as ·17 c.g.s.

are drawn on the sides of the platform so as to prolong the white lines on the sides of the magnetometer. By this means currents in amperes or potentials in volts can be at once read off without any calculation.

The graduation of the current galvanometer may also be performed by means of electrolysis. The electrochemical equivalents of a large number of the metals have been determined, and it is only necessary therefore, in order to graduate the instrument, to join it in series with a proper electrolytic cell and a constant battery, and to compare the amount of metal deposited on the negative plate of the cell with the total quantity of electricity which flows through the circuit in a certain time. A convenient cell may be made with two plates of copper, each about 50 square cms. in area, held parallel to one another about 2 cms. apart, and immersed in a nearly neutral solution of copper-sulphate in a glass beaker. A current of one ampere through such a cell gives a very good result. This current will deposit about 1·2 grammes of copper per hour, which, when light plates are used, can be very accurately weighed. The plates should be carefully cleaned, dried, and weighed before being immersed in the liquid, and care must be taken not to send the current through the cell at all until it is made to flow continuously for the experiment. The instant the current through the cell is completed should be noted, and readings of the current galvanometer taken at equal short intervals during the time allowed for the experiment. The average of these readings is to be taken as the average reading of the galvanometer. When the experiment has been completed the plates are to be taken out and very carefully washed in clean water and dried before being weighed. It is generally better to calculate the quantity of electricity from the gain of the negative plate than from the loss of the positive. The electro-chemical equivalent of copper has been recently determined afresh with great care in the Physical Laboratory of the University of Glasgow by Mr. Thomas Gray, and as the result of his experiments ·000331 of a gramme of copper is deposited by the passage of one coulomb of electricity. Dividing the gain of weight of the negative plate by this number, we obtain the total number of coulombs which have flowed through the galvanometer during the experiment. This divided by the time in seconds gives the average current in amperes; from which the number corresponding to the position of the magnetometer on the platform can be determined, and the graduation completed in the manner described above.

Errata in Part II. of this Article—P. 106, line 2 from top, for $2\pi nCA$ read nCA; line 5, for $CD(=x)$ read $CD(=y)$; line 6, for $DE(=y)$ read $DE(=x)$.

ANDREW GRAY

NORWEGIAN GEODETICAL OPERATIONS[1]

THIS publication of the Norwegian Committee of the Association for the Measurement of Degrees in Europe is a report of the observations made at two tidal stations in Norway—Oscarsborg and Drontheim. The report opens with a summary of the previous tidal operations carried out in Norway. From June 8 to 28, 1835, a series of observations were made at a large number of stations at the request of the English Admiralty. These observations, together with those from numerous other stations in Europe, along the North Sea, and Atlantic coasts, were reduced and compared by the Rev. Dr. Whewell, and were published by him in the *Philosophical Transactions* of 1836.

The next tidal operations recorded were undertaken in 1839, to ascertain whether, as supposed, the Norwegian coast is slowly rising. With this object, marks were cut in the rock at twenty-six points along the coast, and the date was inscribed in each case. On the eastern coast, where there is little or no tide, only one mark was cut, at the level of the water; but along the western and northern coasts two marks were made, one at high water, and the other at low water. The report gives a detailed description of the position of these marks. In 1865 observations were made to ascertain if any alteration had taken place in the relative positions of these marks and of the sea, and it was found that on an average the land was rising at the rate of 3″ in 26 years. It was, however, seen that such determinations were very inadequate, since the mean level of the sea had not been taken into account, and the marks were not connected by lines of levelling. The whole question was accordingly placed before a Committee of the University. This Committee, however, came to no conclusion, principally on the ground of the expense of the necessary levelling operations. It was then pointed out by Prof. Fearnley, the chairman of the Committee of the Association for the Measurement of Degrees in Europe, that the resolutions of the Association at the General Assembly in 1864, at Berlin, had given an increased importance to the determination of the mean tide level. Nevertheless, it was not until 1876 that any steps were taken to establish a series of tidal stations provided with self-registering apparatus, as enjoined by the Association; six stations have now been established, and two more are to be formed.

Previously however, namely, in 1872, two stations had been established, one at Oscarsborg, in the fortified island of Kaholmen, situated in the Christiana fiord, in connection with submarine mining, and where observations have been recorded from 1872 to 1879; the other at Drontheim in connection with the harbour works, the observations at this station have extended from 1872 to 1878. These two sets of observations form, as already mentioned, the subject of the present report.

At both stations self-registering apparatus of simple designs were used. At Oscarsborg the rise and fall of the tide was marked on a *plane* sheet of paper attached to a frame moving horizontally. The motion was imparted to this frame by a weight, which, in order that the motion might be uniform, was connected to a clock. The float was inclosed in a tube, so as to annul the disturbing action of the waves. At Drontheim the paper was attached to a drum rotating uniformly by means of clock-work. The apparatus was placed at the end of a pier, and the scribing pencil was connected by a system of rods to a bridge, the other end of which was supported by a pontoon. In this manner the action of the waves was eliminated. The mean tide level was obtained at these two stations by taking the arithmetic mean of the heights of the tide recorded on the diagrams for each hour, and not by the more accurate method based on the areas described. But since these observations extended over several years, the result is no doubt practically the same; moreover, neither apparatus was provided with any means of obtaining the areas automatically, and to calculate them would have involved an enormous amount of labour.

The observations at both stations are arranged in a series of tables.[1] Table I. gives the heights of the tide at each hour, commencing at noon, for every day during one month.[2] The arithmetic mean of the heights of the tide at the *same* hour each day during the month, are also given for every hour of the day; the moon's influence does not appear in these means, they therefore show the influence of the sun. These means, for every month during which the observations lasted, are collected together in Table IV., and the mean of these means is also given. Table II. gives the height of high and low

[1] Publication of the Norwegian Committee of the Association for the Measurement of Degrees in Europe, Part I. (Christiana, 1882.)

[1] The corresponding tables in each set of observations are denoted by the same number.

[2] This table was not extended further, in order to save space in the Report.